4B-1 Find the volume of the solid of revolution generated by rotating the regions bounded by the curves given around the x-axis.

a) $y = 1 - x^2$, $y = 0$ b) $y = a^2 - x^2$, $y = 0$ c) $y = x$, $y = 0$, $x = 1$
d) $y = x$, $y = 0$, $x = a$ ~~e)~~ $y = 2x - x^2$, $y = 0$ f) $y = 2ax - x^2$, $y = 0$
~~g)~~ $y = \sqrt{ax}$, $y = 0$, $x = a$ h) $x^2/a^2 + y^2/b^2 = 1$, $x = 0$

4~~B~~-2 Find the volume of the solid of revolution generated by rotating the regions in 4B-1 around the y-axis.

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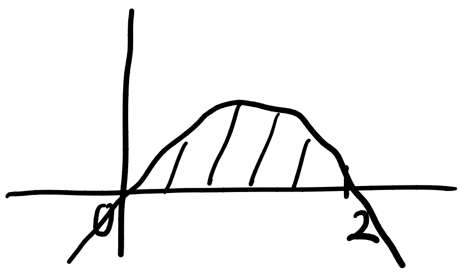
1. e) $y = x(2-x)$, $y = 0$



$$V = \int_0^2 \pi y^2 \, dx$$

$$= \int_0^2 \pi x^2(4 - 4x + x^2) \, dx$$

$$= \pi \int_0^2 x^4 - 4x^3 + 4x^2 \, dx$$

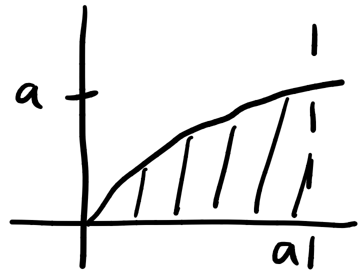
$$= \pi \left(\frac{x^5}{5} - \frac{4x^4}{4} + \frac{4x^3}{3} \right) \Big|_0^2$$

$$= \pi \left(\frac{3 \cdot 2^5 - 15 \cdot 2^4 + 20 \cdot 2^3}{15} \right)$$

$$= \frac{8}{15} \pi (12 - 30 + 20)$$

$$= \frac{16}{15} \pi$$

g) $y = \sqrt{ax}$, $y = 0$, $x = a$



$$V = \int_0^a \pi y^2 \, dx$$

$$= \int_0^a \pi (ax) \, dx$$

$$= \pi a \int_0^a x \, dx$$

$$= \pi a \left(\frac{x^2}{2} \right) \Big|_0^a$$

$$= \frac{\pi}{2} a^3$$

2. e) $V = \int_0^2 2\pi x \cdot y \, dx$

$= 2\pi \int_0^2 x(2x - x^2) \, dx$

$= 2\pi \int_0^2 2x^2 - x^3 \, dx$

$= 2\pi \left(\frac{2x^3}{3} - \frac{x^4}{4} \right) \Big|_0^2$

$= 2\pi \left(\frac{8 \cdot 2^3 - 3 \cdot 2^4}{12} \right)$

$= \frac{\pi}{6} (16)$

$= \frac{8\pi}{3}$

g) $V = \int_0^a 2\pi x \cdot y \, dx$

$= 2\pi \int_0^a x\sqrt{ax} \, dx$

$= 2\pi\sqrt{a} \int_0^a x^{\frac{3}{2}} \, dx$

$= 2\pi\sqrt{a} \left(\frac{x^{\frac{5}{2}}}{5/2} \right) \Big|_0^a$

$= \frac{4}{5}\sqrt{a}\pi (a^{5/2})$

$= \frac{4}{5}\pi a^3$

4C-2 Find the volume of the region $0 \le y \le x^2$, $x \le 1$ revolved around the y-axis.

4C-3 Find the volume of the region $\sqrt{x} \le y \le 1$, $x \ge 0$ revolved around the y-axis by both the method of shells and the method of disks and washers.

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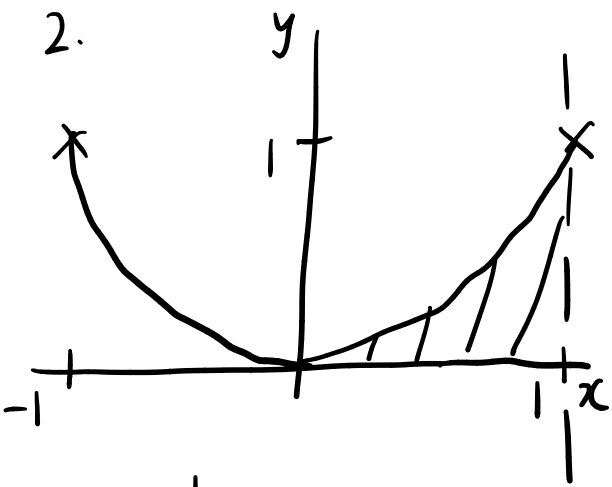
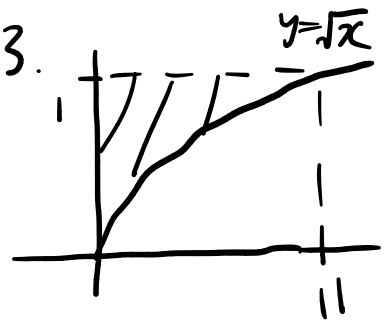
2.



$$V = \int_0^1 2\pi x \cdot y \, dx$$
$$= \int_0^1 2\pi x \cdot x^2 \, dx$$
$$= 2\pi \int_0^1 x^3 \, dx$$
$$= 2\pi \left(\frac{x^4}{4} \right) \Bigg|_0^1$$
$$= \frac{\pi}{2}(1-0)$$
$$= \frac{\pi}{2}$$

3.



Method of shells:

$$V = \int_0^1 2\pi x(1-y)\,dx$$
$$= 2\pi \int_0^1 x(1-\sqrt{x})\,dx$$
$$= 2\pi \int_0^1 x - x^{3/2}\,dx$$
$$= 2\pi \left(\frac{x^2}{2} - \frac{x^{5/2}}{5/2} \right) \Bigg|_0^1$$
$$= 2\pi \left(\frac{1}{2} - \frac{2}{5}(1) \right)$$
$$= \frac{\pi}{5}$$

Method of disks:

$$V = \int_0^1 \pi x^2 \, dy$$
$$= \int_0^1 \pi (y^2)^2 \, dy$$
$$= \pi \int_0^1 y^4 \, dy$$
$$= \pi \frac{y^5}{5} \Bigg|_0^1$$
$$= \frac{\pi}{5}$$

$$(1-y^2)^2$$
$$= 1 - 2y^2 + y^4$$

Method of washers:

$$V = \int_0^1 \pi R^2 - \pi(1-x)^2 \, dy$$
$$= \pi \int_0^1 1 - 1 + 2y^2 - y^4 \, dy$$
$$= \pi \left(\frac{2y^3}{3} - \frac{y^5}{5} \right) \Bigg|_0^1$$
$$= \pi \left(\frac{2}{3} - \frac{1}{5} \right) \times$$
$$= \pi$$

4J-3 A very shallow circular reflecting pool has uniform depth D, and radius R (meters). A disinfecting chemical is released at its center, and after a few hours of symmetrical diffusion outwards, the concentration of chemical at a point r meters from the center is $\dfrac{k}{1+r^2}$ g/m^3.

What amount A of the chemical was released into the pool? (Give reasoning.)

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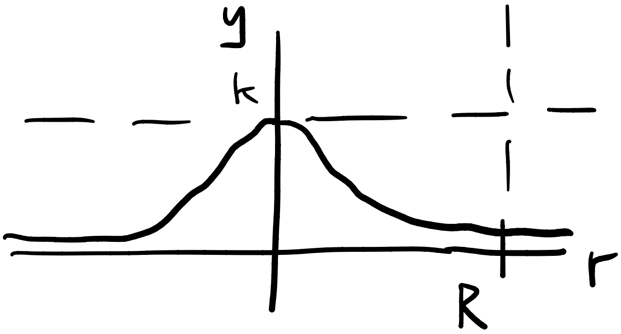

$$y(r) = \frac{k}{1+r^2}$$

$$u = r^2$$
$$du = 2r\,dr$$

$$\Delta V = 2\pi r D\,\Delta r$$

$$u = R^2$$

$$\Delta A = \Delta V \cdot y(r) = 2\pi r D \cdot \left(\frac{k}{1+r^2}\right)\Delta r$$

$$\Rightarrow A = \int_0^R 2\pi r D\left(\frac{k}{1+r^2}\right) dr$$
$$= 2\pi Dk \int_0^R \frac{r}{1+r^2}\,dr$$
$$= 2\pi Dk \int_0^{R^2} \frac{1}{2(1+u)}\,du$$
$$= \pi Dk \ln|1+u|\Big|_0^{R^2}$$
$$= \pi Dk \ln|1+R^2|$$

$\therefore$ $A = \pi Dk \ln|1+R^2|$,

the amount of chemical in an infinitesimal width dr is proportional to its distance r away from the origin.

In the limit, the infinitesimal amount is the infinitesimal volume multiplied by its concentration of that shell a distance r away from the origin.

Taking the integral of that over the radius R of the cylinder gives the total amount.